Turing Machines as Conscious Computing Machines

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Abstract. We uncover certain universal features of Turing machines (TM) as operating in a perpetually changing environment which can have sudden and highly random influence on TMs themselves. TM adapts and assimilates the changed environment and performs its computational functioning in new conditions. We show that this transcends essentially Turing computability relative to a ground model of ZFC. We distil the formal counterparts responsible for the adaptation and assimilation of TM and propose they may underlie the conscious behaviour of general systems including living creatures. However, in this last case more work leading to layer's structure of TMs is needed. We also make an attempt toward social TMs by finding the way how TMs can group and cooperate.

Keywords: Turing machines in models of ZFC, forcing, formal aspects of consciousness.

1 Introduction

A Turing machine is a formal concept explaining what any computability process looks like. The usual way of seeing computability as a formal processes is rooted in arithmetical constructions and rather lacks of the broader outer, i.e. environmental, perspective. We want to fill the gap and extend formally defined TM over external formal environment. The environment reacts on such TM in a random way and this modifies TM. The implementation of the modification will not destroy TM but rather enlarges its computational abilities. We base our analysis on the well-known relation of the axiomatic Zermelo-Fraenkel set theory with the axiom of choice (ZFC) with its models (e.g. [1]). The dynamics of models (forcing extensions) is the factor representing the reaction of TM on the environmental stimuli. From the point of view of pure ZFC (lacking the perspective of models) the dynamics is out of the reach. It is also widely known fact that despite the simplicity of TM, it is implemented in a way in any software of any classical computer. Similarly the extension of TM over the formal environment (FE) (represented by ZFC tools) we work out here, being direct and simplifying, still it leads to universal features.

One important aspect of the approach is presented by the mechanism of assimilation of the FE reaction on TM such that this TM is changing from the state of completely random affecting the outer stimuli into the modified TM with the stimuli becoming its building blocks. The entire process resembles a way how a living organism is gaining its skills under the influence of the perpetually changing environment. The formal perspective proposed here seems limiting at first sight, but it might bear universal mathematical features so that they may underlie the conscious behaviour of creatures in the world. This is quite analogously to the universal TM which serves as universal computational machines in the class of all classical computers (TMs) (e.g. [2]) even though this reduction to the universal TM is not evident in each case.

2 Key terminologies

Before we present the main construction, in this section we grasp together basic facts regarding TM as well as TM with oracles (o-TM), Turing uncomputable classes and algorithmic randomness, and set theoretic constructions like forcing. They will be needed in the following sections.

2.1 Classical Turing machines with oracles

Let Q be a finite set of possible (internal) states of TM, $Q = \{q_0, \ldots, q_n\}, n \geq 1$; let t be an infinite two-sided tape containing cells. Each cell has written in the symbol 1 or the symbol B, blank, $S = \{1, B\}$. Let h be a reading head which in each single step can read the content of a cell on the tape t and move to the right, if $X = R$, or to the left, when $X = L$. TM can also rewrite (change) the existed symbol $s \in S$ in the cell to a new symbol $s' \in S = \{1, B\}$.

Definition 1 ([2]). *TM comprises of the tape t, head h, and TM is in one of its internal states from S. The functioning of TM is described by the collection of steps governed by the collection of symbols (q, s, q', s', X), where $q, q' \in Q, s, s' \in S, X \in \{L, R\}$, such that TM in the state s reads the symbol s from the tape and changes its state to s' and writes down in the scanned cell a new symbol s' and moves its head to the right, if $X = R$, or to the left, if $X = L$, by a single cell in each step.*

Thus, the operation of any TM is governed by the *partial* function (not everywhere defined, since TM may not produce any writing over a cell content and move of the head):

$$Q \times S \to Q \times S \times \{R, L\}.$$

The Turing program is a finite set of the values assigned to (q, s, q', s', X), i.e. it is a finite set of quintuples. The states S contains the leftmost state q_0 which is '1' in the cell (initial state) and the halting state (which, however, may not be attained). The detailed discussion of how TM computes, based on the above definition, can be found, e.g., in [2].

Definition 2. *TM with oracle $A \subset \mathbb{N}$, o-TM or TM^A, it is a TM with an additional infinite read-only tape A, on which there is written the characteristic function of A ($\{0,1\}$-binary sequence). The reading of the tape A is the part of the functioning of TM.*

Thus, the o-TM allows for performing computations on the data, which might not depend on any action of any TM. Given the o-TM and putting the set uncomputable but computably enumerable (c.e.) into the oracle, one gets entire hierarchy of uncomputable Turing classes. The example are $\emptyset$ sets expressing the halting problem of TM, and their n-th uncomputable jumps $\emptyset^{(n)}$ placing $\emptyset^{n-1}$ into the oracle. We refer the reader to the excellent exposition by Robert Soare [2] or in [3].

2.2 Arithmetic and Turing classes

The concept of algorithmic randomness is based on the hierarchy $\Sigma^0_n, n = 0, 1, \ldots$ of complexity of arithmetic formulas (e.g. [2–4]). Higher arithmetic classes of formulas correspond to objects, which can be determined by a TM, however, with the increasing computational complexity. So to define a purely random binary infinite sequence $\sigma \in 2^\omega$, one requires that σ omits all or some classes. This is how the original Martin Löf (ML) test for randomness arose.

1. ML test: A sequence $\{A_n, n \in \mathbb{N}\}$ of uniformly computably enumerable (c.e.) (i.e. c.e. together with the set of its indices [3, p.11]) of $\sum^0_1$ classes ($\sum^0_1$ subsets of sequences from 2^ω) such that $\forall_{n\in\mathbb{N}}\ (\mu(A_n) < 2^{-n})$.

2. $A \subset 2^\omega$ is ML-null, when there exists a ML test $\{A_n, n \in \mathbb{N}\}$, such that $A \subseteq \bigcap_{n\in\mathbb{N}} A_n$.

3. $\sigma \in 2^\omega$ is ML-random, if $\{\sigma\}$ is not ML-null (for each ML test).

4. A ML test $\{A_n, n \in \mathbb{N}\}$ is *universal*, when $\bigcap_{n\in\mathbb{N}} B_n \subset \bigcap_{n\in\mathbb{N}} A_n$ for all ML-tests $\{B_n, n \in \mathbb{N}\}$.

Lemma 1. *There exists a universal ML test.*

ML-random sequence $\sigma \in 2^\omega$ is known to be 1-random. The direct modification to Σ^0_n classes gives rise to the hierarchy of n-random sets, for all $n \geq 1$.

i. ML_n test: A sequence $\{A_k,\ k \in \mathbb{N}\}$ of uniformly c.e. of $\sum^0_n$ classes ($\sum^0_n$ subsets of sequences from 2^ω), such that $\forall_{k\in\mathbb{N}}\ \big(\mu(A_k) < 2^{-k}\big)$.

ii. $A \subset 2^\omega$ is ML_n-null, when there exists a ML_n test $\{A_k,\ k \in \mathbb{N}\}$, such that $A \subseteq \bigcap_{k\in\mathbb{N}} A_k$.

iii. $\sigma \in 2^\omega$ is n-random if $\{\sigma\}$ is not ML_n-null (for each ML_n test).

Since the set of all subsets of natural numbers represents real numbers, the usual way how the sets of all reals are represented in models of set theory is 2^ω (or ω^ω), which is a Polish space [1, 4]. That is why speaking about reals in models of ZFC is speaking about infinite binary sequences.

2.3 Forcing in set theory

Given B a complete Boolean algebra in a model M of ZFC, we have:

Lemma 2. *There exists a generic extension $M[r] \supsetneq M$ iff B is atomless in M.*

Definition 3. *The measure algebra (random algebra) is the Boolean algebra B which is the algebra of Borel subsets of $\mathbb{R}$ modulo the ideal of subsets of Lebesgue measure zero, $B = Bor(\mathbb{R})/\mathcal{N}$.*

Lemma 3. *The measure algebra B is the atomless complete Boolean algebra.*

It follows that there exist nontrivial random real numbers $r \in M[r] \neq M$ whenever B is the measure algebra in M.

3 Results

As we have already noticed, the oracle TM, $\mathrm{TM}^A, A \subset \mathbb{N}$, leads to the entire spectrum of Turing uncomputable classes. Starting with A as certain c.e. set, which is not Turing computable, TM^As compute the characteristic classes of other sets, belonging to the same Turing class as A itself. Then taking higher Turing classes as oracles, we repeat the computability by TMs with this oracle and so on. The idea behind finding counterparts of conscious behaviour of operating TMs is based on the following basic observations

A. Consciousness reflects self-orientation and self-understanding of a system as being in the random outer environment.
B. The environment acts on the system by random stimuli.
C. The stimuli can change the system and the changes are assimilated by it.
D. The system understands the changes and then the assimilated stimuli are no longer random or alien.
E. The effect of the stimuli on the system can be more focused and then it means a stress or it can be less focused and then it means a satisfaction. Stress and satisfaction are understood as two basic emotions. They can change the system and the changes are assimilated by it, too. They are no longer random, as well.

We try to find canonical and formal counterparts of the above points in the realm of calculational processes within TMs. First, we need to understand what can be taken as outer environment for any TM. The point is that TM is an arithmetical concept, but when for higher Turing classes TM extends Peano arithmetic (PA) in a sense, o-TMs produce also independent of PA axioms functions. That is why we propose to consider any o-TM as naturally embedded in the axiomatic ZFC theory. However, to reflect randomness of the outer stimuli coming from ZFC we propose to base our consideration on the Martin Löf notion of randomness, or more precisely, on a weaker its form, i.e. Solovay generic randomness of infinite binary $\{0, 1\}$ sequences, e.g. [3]. This Solovay randomness is a 'miniaturisation' to arithmetic of the broader concept of randomness genericity in ZFC [3]. This extended to ZFC notion of randomness has been introduced also by Solovay and is known as a forcing in set theory, which we have briefly discussed in the previous section. This last we call a ZFC-randomness and it is the proper notion for our external to the TM environment. Thus concluding, we are choosing ZFC axiomatic set theory as a formal environment for o-TMs. But this is merely the first approximation since PA independent statements can be also ZFC independent: ZFC does not prove or disprove them but they and their negations are rather consistent with ZFC. This last statement means that there is a model of ZFC where p is true and the other model where $\neg p$ is true, and both models have all provable in ZFC propositions as their true statements. That is why the method of forcing is especially well-suited for such situations. But if so, we should extend the ZFC axiomatic environment over models of ZFC, where their difference is the valid ingredient of the approach. This is precisely what we are doing when searching for the proper formal external to the TM environment.

Definition 4. *o-TM interacting with the external ZFC environment,* $(o\text{-}TM)_M$*, is the ordinary o-TM defined in a standard transitive model* M *of ZFC.*

Remark 1. Since ZFC interprets PA, so the constructions of TM are expressible in models of ZFC. The model M above is not specified at this place; one can take as M some countable transitive model (CTM) or V – the entire universe of sets, or some internal model, or others. We will discuss briefly the distinctions between the choices in what follows.

Remark 2. Given a ZFC model M, it is generally possible to add new real numbers to it. In the case of a CTM M one can add even a continuum of many different reals by nontrivial forcings from the outside of M. This generally follows from the relation between reals in V (let it be $\mathbb{R}$) and the reals in M: $R_M \subset \mathbb{R}$ and $|R_M| = \omega$ in V. The Remark 1 below explains the definition which follows.

Remark 3. TMs in different standard transitive models of ZFC with the standard natural numbers object are equivalent in the sense that PA + ZFC are equivalent in the models. The nonequivalent inputs, which extend the models and Turing computability or ZFC, can appear in oracles.

Definition 5. *The external ZFC environment interacts with* $(o\text{-}TM)_M$ *by adding reals into the oracles or by non-generic oracles.*

Let N be the universe of sets (e.g. CTM) for $(o\text{-TM})_N$ and M for $(o\text{-TM})_M$. If N is a ground model for M, i.e. $N[s] = M$ for generic s, we say N is the shrinked version of M due to the stimuli $r_s \in M \subset M[r]$. Similarly, $M[p]$ is the extended version of M due to the stimuli p generic for M. Then

Definition 6. r_s *is a stress stimuli for* $(o\text{-}TM)_M$ *and* p *is the satisfaction stimuli for* $(o\text{-}TM)_M$*. The resulting states of* $(o\text{-}TM)_M$ *, i.e.* N *and* $M[r]$ *are called stress and satisfaction states respectively. The* M *resulting in* M *(without nontrivial changes) means a neutral state, see [13].*

Remark 4. Note that given two different $(o\text{-TM})_{M_1} = \text{TM}_1$ and $(o\text{-TM})_{M_2} = \text{TM}_2$, their 'social relations' can be also given in terms of the interactions of the external environment, since the part of this is each TM with respect to the other. In particular TMs could react on the emotions each to the other by oracles.

The reaction of $(o\text{-TM})_M$ can be neutral (no reaction) or active, i.e. the oracle A_M becomes extended by reals in the extended by forcing model $M[r]$, i.e. $A_{M[r]}$. Thus, $A_{M[r]}$ contains generic reals.

Now we can confront the oracle TM, interacting with the external domain with the conditions A. – E. from the beginning of this section.

Theorem 1. *Let $M \to M[r]$ be the random forcing, adding the real r to M. There is a canonical formal way in which $(o\text{-}TM)_M$ fulfils conditions A. – E.*

Proof. Regarding A. that $(o\text{-TM})_M$ reflects 'self-orientation and self-understanding of itself as being in the random outer environment'. This is in terms of Turing machines augmented by the external interaction with the environment as in Definition 5. 'Understanding' by TM is due to the ZFC, realised in the model M, where there are in use by TM internal to M real numbers, R_M. So the space of states of this TM includes also R_M. A real $r \in M[r]$ is not in M, but it will be inserted into the oracle. It is random for M by the forcing and since it is not predictable by M itself (i.e. which random real it will be). At this stage, M thinks there are *all* real numbers in M (according to understanding given by ZFC). After r is included into the oracle, TM assimilates it and changes its state to $M[r]$ so thus TM now considers r as valid real number since $R_M \subsetneq R_{M[r]} \subset \mathbb{R}$. Regarding self-orientation, this is also connected with assimilating external environmental reals as parameterising the external space. We will explain it in the Example below.

Regarding B., this is precisely stated in Definition 5.

Regarding C., the assimilation property has been already explained above as adding random r to M. The change of the state follows as $M \to M[r]$ and the state of TM after assimilation is $M[r]$.

Regarding D. that TM 'understands the changes and then the assimilated stimuli are no longer random or alien', it has been already indicated at A. above, where understanding has been given by the process (following the change of the state of TM) from ZFC_M to $\text{ZFC}_{M[r]}$. When M is in the state $M[r]$, r is no longer random (still there can be new random reals $r' \in \mathbb{R}$ to $M[r]$, $r' \notin M[r]$ and $r' \in M[r][r']$).

Regarding E., the stimuli of 'moderate focus' (a satisfaction) effects the random forcing extension $M \to M[r]$ while this of 'high focus' (a stress) results in taking a ground model N for M, i.e. $N[r] = M$, and the stimuli is not absorbed by N. The satisfaction is connected with the expansion and extension of the model while stress with the shrinking of it (see the discussion about the multiverse in the end of this section).

One could wonder whether the external stimuli which can be of arbitrary high degree of randomness can be assimilated by our TM. Let $\sigma \in 2^\omega$ be an arbitrary (of arbitrary high degree of algorithmic ML randomness) subset of $\mathbb{N}$ in V.

Proposition 1. *For any σ as above, there exists (o-TM)$_M$ with the M-random $r \in M[r]$ in the oracle which can reflect the degree of randomness of σ.*

Proof. This is based on fundamental facts from algorithmic randomness. First, randomness in arithmetic is the instance of ZFC Solovay forcing when 'miniaturised' to PA [3]. It means that we make the forcing procedure in PA theory without bothering of ZFC properties of the sequences $\sigma \in 2^\omega$. Any ML 1-random binary sequence σ_1 omits the Σ^0_1 subsets of 2^ω of arbitrary small Lebesgue measure (ML test). From the other side, given the random real r with respect to ZFC model M, r omits all measure zero subsets of $(2^\omega)_M$ which means that such r is also arithmetically 1-random with respect to M. Thus, knowing r be generic random in M, it is 1-random with respect to the pair $(M, M[r])$ $(r \notin M)$. Given higher $n > 1$ ML random σ_n, it omits every subset of 2^ω of arbitrary small measure and thus a M-random r omits every measure zero subset of 2^ω coded in M. This last certainly omits every n-arithmetic subset of reals with zero measure in M; thus, such a sequence is n ML random with respect to the pair $(M, M[r])$. So, Solovay generic r can indeed reflect in the pair $(M, M[r])$ the arbitrary high degree of randomness.

Remark 5. Assuming that M be the so-called ω model of ZFC, i.e. a transitive standard one with the standard natural numbers object $\mathbb{N}$, one obtains the minimal ZFC driven discrepancies between TM in M and in V. The discrepancies can be valuable by themselves, however, we do not delve in it here.

Remark 6. The above proposition works as far as there exists the generic filter of the Boolean algebra B in M. Otherwise $r \in M$ and it can not omit all measure zero subsets of 2^ω. It is known that this is always the case (generic r exists) for countable transitive models M. However, in the universe of sets V there does not exist any generic ultrafilter, hence random r as well. The usual solution is to build the Boolean model V^B in V, with the canonical embeddings $V \subset V^B \subset V$, and prove in V^B that with the value 1 there exists random real r (hence, a generic ultrafilter). This r again omits *all* measure-0 subsets of $\mathbb{R}$ in V^B with the value 1. Thus, we can assume that a random real r exists in M and V and it is n-random for $n \in \mathbb{N}$. In the multiverse approach – which will be discussed below – there is the family of models closed on the extensions and taking ground models so thus generic random rs always exist for the models.

The importance of the Proposition 1 is that (o-TM)$_M$ can assimilate arbitrary random incomes which appear in the oracle by the response to the external stimuli from V. After the assimilation the final state of TM is externally modified such that it is TM in $M[r]$ and this r is not any longer random in $M[r]$. The process how TM undergoes the changes and perceives them (refers to) from the new state is very important and requires a deeper clarification. Let us augment the Definitions 2 and 4 as

> *The state space Q for (o-TM)$_M$ contains the symbols for the forcing extensions of M, i.e. $|1|$ for $M[r]$ and $|0|$ for M. Whenever any change of M does not occur (trivial forcing, no external stimuli), the state of*

TM remains unaltered, i.e. M, and the state of TM is $|0|$. Thus, if the nontrivial forcing adding r into the oracle took place the state is recognised as $|1|$. The $|0|$ state is assigned also to the shrinking model M to N, since no generic r is added.

*Example 1 (*The modification of TM*).* Let M be a countable transitive standard model of ZFC with the standard natural numbers object. All ZFC provable statements holds true in M, but also Peano arithmetic is derivable from ZFC, so that in M there holds true the ZFC arithmetic statements. Let the external to the TM 3-dimensional spatial domain be parameterised by reals $\mathbb{R} \in V$, so the spatial external domains U are (open or not) subsets of $\mathbb{R}^3$. $\mathbb{R}$ contains both, reals R_M from a general ground model M and reals which are not in M. Among them there are generic reals with respect to all possible random forcings over M (and for other forcings of course) and new reals which are not generic and are not in M. The spatial domain, where internal TM acts (from the point of view of TM), is parameterised by R^3_M. From the outside (from the V point of view), R_M is countable, though from the M point of view, M contains *all* reals. However, the possible generic reals (with respect to various random forcing extensions) is continuum many from the external point of view, so the probability to find a generic real in $\mathbb{R}$ is much higher than for nongeneric. Let the external stimuli be generated in V and represented by some generic to the M real r. The interaction of TM in M with this stimuli leads to the overwriting on the oracle tape the binary representation of r. This r is not in M, however, the TM state is fixed to $|1|$ and TM after the entire process is internal to the forcing extension $M[r]$. At this final stage, r is no longer random in $M[r]$. The assimilation of the external random stimuli is completed. By the same process the spatial orientation can now be gained by identifying the stimuli r with the point corresponding to the external parameterisation.

This is a quite nontrivial task to decide for TM whether its actual state is $|0|$ or $|1|$. The reason is that ZFC and PA are theories in the first-order language and as so their provability power does not allow for 'seeing' the set models of the theories (otherwise they would prove their consistency). Moreover for a CTM $M[r]$ this is always the ground model for a subsequent random forcing leading to $M[r][s]$ and so on. Still, we can assume that the interaction with the external environment gives the information about the state, e.g. a random r for M being assimilated by the oracle of TM loses its randomness and indicates the state of TM is now $|1|$. Another possibility is to refer to general results concerning a definability of the ground model M in the extension $M[r]$ (e.g. [5]). We do not elaborate on this important issue here but rather it will be addressed elsewhere. Let us resume this as: $(o\text{-TM})_M$ is in the $|1|$ state means that the oracle has been just added (in the last step) as a random real r coming from the external stimuli and there exists a model of ZFC N such that $N[r] = M$. $(o\text{-TM})_M$ is in the $|0|$ state meaning that in the last step there is no random r extending the oracle.

Given introduced TM as carrying some basic features of conscious-like behaviour, we would like to see this phenomenon more broadly. Especially, are there certain formal counterparts, already at this very basic level, which would indicate group- or 'social-like' activities of several such defined TMs? Again, guiding principles come from studying models of ZFC in this context.

As follows from the discussion above, the internal $(o\text{-TM})_M$ to M carries among its states the information about actual random forcing extensions $M \to M[r]$. However, for CTMs 'to be extended by a random forcing' is generic, i.e. it is always possible to make yet another such extension starting from $M[r]$ and this seems to be a fundamental feature of TMs. This phenomenon is deeply rooted in the foundations of set theory. One approach to set theory is based on a distinguished universe of sets, like V, which is the class containing all sets, the other approach is a set theory without the specific choice of the basic universe of sets. The first will be marked as U and the second as MV – multiverse, in what follows. Given a CTM model M, its multiverse is the family of models containing M closed with respect to taking all forcing extensions and all ground models of its members. The concept has appeared as very fruitful (e.g. [6]) and it has been shown leads to different truths values for set theory statements which can be proved in V and in all models in MV (e.g. the continuum hypothesis is true in the generalized to inner models MV [7]). The point is that the MV approach is based on the scattered truth concept depending on models of ZFC. The U approach is based on the centralised truth with respect to the distinguished universum of sets. The former one is more close to the decentralised nets point of view (scattered notion of truth) and this is attractive also for TMs interacting with the external environment and interacting with other TMs in the net.

In the context of $(o\text{-TM})_M$ let M be a CTM of ZFC and this TM changes the models along with the external random stimuli and the extension adapts this TM to the new random condition. The state space for such TM contains the positioning of TM in the *random* MV, rMV, for M. In fact this positioning is merely local, i.e. the actual pairs $(N, N[r])$ enter the game for the current state of TM. We do not require that TM is capable for any identification of models N in the entire structure rMV. Still rMV represents the space of possible paths for $(o\text{-TM})_M$ when it interacts with the environment. From the perspective of such TM it does not know about possible external V, rather rMV creates the entire 'universe' of sets. Quite similar as MV replaces a single universe of sets. Taking rMV and V simultaneously and allowing for the interactions is the place where conscious phenomena can enter the stage in the model presented here.

Now given several $(o\text{-TM})_{M_i}, i = 1, 2 \ldots$ and taking two of them, it can happen that their momentary models coincide, $M_i = M_k$ or not. More generally there can exist (or can not) a model M_{ij} containing both M_i, M_j as submodels. In general zig-zag moves (taking extensions or grounds) within the structure rMV leads to building the entire net of connections between states of M_i and M_j. Another factor in creating nets is the nonamalgamation property, i.e. two forcing extensions of a CTM model are not submodels of a common model of the same hight (e.g. [8]).

The power of oracle TM computability can be also directly seen in the case of forcing [9]. It has been proved that for the oracle, Gr_0, which would be the elementary graph of M (the set of true ZF statements in M) plus the forcing partial order $\mathbb{P}$ in M, TM^{Gr_0} computes the forcing extension $M[r]$. Which indeed means that building the connections between TMs by forcing, inherently relies on the oracle Turing computability. More precise understanding of the impact of the above formal elements on true functioning of TM or nets of TMs requires much further studies also conceptual in the foundations of science.

4 Discussion

We have introduced the Turing machine interacting with the external environment, and shown formal counterparts which could be related to, if not underlie, the conscious behaviour of the real systems in our world. 'Exterior to TM' means not only as situated in different spatial regions, but also separated by different mathematics. We have shown that when TM lives in the set-theoretic world, based on the multiverse paradigm, and it is confronted with the external environment organised by the single set theoretic universe V, then the contact region of both may be the carrier of certain phenomena, allowing for developing conscious relations to the world. We think, though did not present a full justification for it here, that the structure is universal (rooted in foundations of mathematics) also for systems in real world showing conscious reactions.

The situation where one pays a bigger attention to forcing relations than to sets themselves, resembles to some degree the replacing objects by arrows – morphisms in category theory (cf. [10, 12]). This kind of thinking with the priority of 'forcings over sets' became fruitful also in the context of certain fundamental problems in physics (e.g. [11, 4]). This certainly requires more thorough studies and effort and partial results will be a topic for our forthcoming publication.

Also, the case of living conscious organisms could be approached from the proposed here perspective, even though it looks very simplified at first sight. One option is to introduce the structure of interacting layers. An extension of the formalism over emotions or various social phenomena is the matter of further work. Anyway one can note that the 'random' nature of some social phenomena can be embedded in the system presented here where randomness is coded formally. This is in the sharp opposition to certain previous misconceptions, like considering consciousness as a formal computational model of self-reference and claiming that formal methods would not allow the embedding.

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